FUNCTIONAL DATA ANALYSIS
SPRING 2018

ASSIGNMENT–4

- **Problem 1.** Recall the functional regression model for scalar response:

$$y_i = \alpha_0 + \int x_i(t)\,\beta(t)\,dt + \epsilon_i \tag{1}$$

(1) Use a basis expansion to represent $\beta(t)$ in terms of known basis functions, and estimate $\beta(t)$.
(2) Repeat the computations above after incorporating the roughness penalty as discussed in the lecture.

- **Problem 2.** Continuing with the same setup as above, invoke a further simplification of the functional model by using the basis obtained by way of performing the functional PCA to represent the covariates as:

$$x_i(t) = \overline{x}(t) + \sum_{j \geq 0} c_{ij}\,\xi_j(t),$$

Thus, allowing us to write (1) as

$$y_i = \alpha_0 + \sum_{j \geq 0} c_{ij} \int \xi_j(t)\,\beta(t)\,dt + \epsilon_i \tag{2}$$

Now use a basis expansion to represent $\beta(t)$ in terms of known basis functions, and estimate $\beta(t)$.

- **Problem 3.** Recall the analysis performed in the usual regression analysis to assess the goodness of estimates. Define

$$SSE = \sum_{i=1}^{N} \left(y_i - \int x_i(t)\,\widehat{\beta}(t)\,dt \right)^2$$

$$SSY = \sum_{i=1}^{N} (y_i - \overline{y})^2$$

Write the test statistic explicitly for the cases discussed in the problems above.

- **Problem 4.** The functional regression model for functional response is given by:

$$y_i = \beta_0(t) + \sum_{j=1}^{Q} x_{ij}(t)\,\beta_j(t)\,dt + \epsilon_i \tag{3}$$

(1) Use the fitting criterion $LMSSE$ defined in the lecture to compute explicitly an estimate for $\{\beta(t)\}_{j=1}^{Q}$.
(2) Reflect upon the corresponding statistical tests to test the significance of estimates.