

Name:.....

## FDA

### Assignment 1

#### Bases of function, orthonormalization of functions

Assignments constitute part of the examination and must be handed in time. You are asked to hand in the solutions during a week following the week on which the assignment has been discussed in classes. You can submit either an electronic copy or a hard copy of your work. In the latter case, staple your solutions together.

### **Problem 1 – Gram-Schmidt orthonormalization**

In the lecture, a recurrent procedure for orthonormalizing a sequence of vectors in any Hilbert space was presented and called Gram-Schmidt orthonormalization procedure.

1. First, check that the first two vectors produced are indeed orthonormal.
2. Use then the induction argument to show that the entire procedure produces an orthonormal sequence of vectors (is there any assumption needed for the procedure to succeed – have a discussion).

## Problem 2 – Orthogonalizing monomials

Consider the monomials  $x^m$ ,  $m = 0, 1, 2, \dots$  on  $[0, 1]$ .

1. Produce the first six elements of the Gram-Schmidt orthonormalization using the definite integrals for the polynomials.
2. Compare these with those given in the lecture.

**Problem 3 – Orthogonalizing arbitrary functions** The presented method of orthonormalization is applicable to arbitrary collection of functions within given Hilbert space.

1. Choose arbitrary six functions on interval  $[0, 1]$  and produce the output from G-S procedure either using explicit integrals or assist yourself with the numerical evaluation of the inner products.
2. Take another arbitrary function and approximate it by the orthonormal functions.
3. Produce supporting plots.